Review for the Final Exam:

The final exam will be on November 25th, from 11:45 to 2:00pm in CS&T CNTR 220 (Loop Campus). The exam is a closed book exam; students are allowed to bring one single page of notes and a calculator.

The following topics are included in the final exam. Please read both the lecture notes and the corresponding sections from the textbook:

Chapter 4:
- Randomness (Section 4.1)
- Probability Models (Section 4.2)
- Random Variables (Section 4.3)
- Means and Variances of Random Variables (Section 4.4)

Chapter 5:
- Sampling Distributions for Counts and Proportions (Section 5.1)
- Sampling Distributions of a Sample Mean (Section 5.2)

Chapter 6:
- Confidence Intervals (Section 6.1)
- Tests of Significance (Section 6.2)
- Use and abuse of tests (Section 6.3)

Chapter 7: Inference for the mean of a population (Section 7.1, pages 491-505)
Chapter 8: Inference for proportions (Section 8.1, pages 575-577)

I recommend the following things:
- Review the problems solved in class
- Review the examples from the textbook
- Review Homework 4, 5 & 6
- Solve the recommended problems from below:

Problem 1:

A psychologist studied the number of puzzles subjects were able to solve in a 5 minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a subject. The psychologist found that X had the following probability distribution:

<table>
<thead>
<tr>
<th>Value of X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The probability that a randomly chosen subject completes at least three puzzles in the 5 minute period while listening to soothing music is:
- (A)0.3
- (B)0.4
- (C)0.6
- (D)0.9
The probability that a randomly chosen subject completes at most two puzzles in the 5 minute period while listening to soothing music is:
(A)0.3  (B)0.4  (C)0.6  (D)0.9

Problem 2:

A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6% with the additional revenue going to education. Let \( \hat{p} \) denote the proportion in the sample that say they support the increase. Suppose that 40% of all adults in Ohio support the increase.

i. The mean \( \mu_{\hat{p}} \) of \( \hat{p} \) is:
(A)5%  (B)40% ±5%  (C)0.4  (D)600

ii. The standard deviation \( \sigma_{\hat{p}} \) of \( \hat{p} \) is:
(A)0.4  (B)0.24  (C)0.016  (D)0.00016

iii. How large a sample would be needed to guarantee that the standard deviation \( \sigma_{\hat{p}} \) of \( \hat{p} \) is no more than 0.01?
(A)100  (B)1000  (C)1500  (D)2400

iv. The probability that \( \hat{p} \) is more than 0.50 is:
(A)less than 0.0001  (B)about 0.1  (C)0.4602  (D)0.5

Problem 3:

A random sample of size 25 is to be taken from a population that is normally distributed with mean 60 and standard deviation 10. The average \( \bar{x} \) of the observations in our sample is to be computed. The sampling distribution of \( \bar{x} \) is
A)normal with mean 60 and standard deviation 10
B)normal with mean 60 and standard deviation 2
C)normal with mean 60 and standard deviation 0.4
D)normal with mean 12 and standard deviation 2.

Problem 4:

I take a SRS of size \( n \) from a population that has mean 80 and standard deviation 20. How big should \( n \) be so that the sampling distribution of \( \bar{x} \) has the standard deviation 1?
A) 400
B) 20
C) Approximately 5
D) Cannot be determined unless we know the population follows a normal distribution
Problem 5:
A 90% confidence interval for the mean $\mu$ of a population is computed from a random sample and found to be (9-3, 9+3). Which of the following could be 95% confidence interval based on the same data?
A) (9-2, 9+2)
B) (9-3, 9+3)
C) (9-4, 9+4)
D) Without knowing the sample size, any of the above answers could be 95% confidence interval

Problem 6:
You measure the heights of a random sample of 400 high school sophomore males in a Midwestern state. The sample mean is $\bar{x} = 66.2$. Suppose that the heights of the population of all high school sophomore males follow a normal distribution with unknown mean $\mu$ and standard deviation $\sigma = 4.1$ inches.

i. A 95% confidence interval for $\mu$ is:
A) (58.16, 74.24)
B) (59.46, 72.94)
C) (65.8, 66.6)
D) (65.86, 66.54).

ii. Suppose I had measured the heights of a random sample of 100 sophomore males rather than 400. Which of the following statement is true?
A) The margin of error for our 95% confidence interval would increase.
B) The margin of error for our 95% confidence interval would decrease.
C) The margin of error for our 95% confidence interval would stay the same, because the level of confidence has not changed
D) Standard deviation would decrease.

Problem 7:
Suppose that the population of the scores of all high school seniors that took the SAT-M (SAT-Math) test this year follows a normal distribution with mean $\mu$ and standard deviation $\sigma = 100$. You read a report that says, “On the basis of a simple random sample of 100 high school seniors that took the SAT-M test this year, a confidence interval for $\mu$ is (512-25.76, 512+25.76).” The confidence level for this interval is
A) 90%
B) 95%
C) 99%
D) more than 99.9%
Problem 8:
The heights of young American women, in inches, are normally distributed with mean \( \mu \) and standard deviation \( \sigma = 2.4 \). I select a simple random sample of young American women and measure their heights. The four heights, in inches, are
63    69    62    66
Based on these data, a 99% confidence interval for \( \mu \), in inches, is
A) (65-1.55,65+1.55)
B) (65-2.35,65+2.35)
C) (65-3.09,65+3.09)
D) (65-4.07,65+4.07)

Problem 9:
The heights of young American women, in inches, are normally distributed with mean \( \mu \) and standard deviation \( \sigma = 2.4 \). If I want the margin of error for a 99% confidence interval to be \( \pm 1 \) inch, I should select a random sample of size:
A) 2 B) 7 C) 16 D) 39

Problem 10:
In tests of significance about an unknown parameter, the test statistic
A) is the value of the unknown parameter under the null hypothesis
B) is the value of the unknown parameter under the alternative hypothesis
C) measures the compatibility between the null and alternative hypotheses
D) measures the compatibility between the null and data

Problem 10:
In the last mayoral election in a large city, 47% of the adults over the age of 65 voted Republican. A researcher wishes to determine if the proportion of the adults over the age of 65 in the city who plan to vote the Republican in the next mayoral election has changed. Let \( p \) represent the proportion of the population of all adults over the age of 65 in the city who plan to vote republican in the next presidential election. In terms of \( p \), the researcher should test which of the following null and alternative hypotheses?
A) \( H_0: p = 0.47 \) vs. \( H_a: p > 0.47 \)
B) \( H_0: p = 0.47 \) vs. \( H_a: p \neq 0.47 \)
C) \( H_0: p = 0.47 \) vs. \( H_a: p < 0.47 \)
D) \( H_0: p = 0.47 \) vs. \( H_a: p = 0.47 \pm 0.03 \), because 0.03 is the margin error for most polls.

Problem 11:
In testing hypotheses, which of the following would be strong evidence against the null hypothesis?
A) using a small significance level
B) using a large significance level
C) obtaining data with a small p-value
D) obtaining data with a large p-value
Problem 12:
In a statistical test of hypotheses, we say the data are statistically significant at level $\alpha$ if:
A) $\alpha = 0.5$
B) $\alpha$ is small
C) the p-value is less than $\alpha$
D) the p-value is greater than $\alpha$

Problem 13:
In a statistical test of hypotheses, the p-value tells us
A) if the null hypothesis is true
B) if the alternative hypothesis is true
C) the largest level of significance at which the null hypothesis can be rejected
D) the smallest level of significance at which the null hypothesis can be rejected

Problem 14:
The time needed for college students to complete a certain paper-and-pencil maze follows a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the mean time $\mu$ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the maze. It takes them an average of $\bar{x} = 31.2$ seconds to complete the maze. Use this information to test the hypotheses $H_0: \mu = 30$ $H_a: \mu \neq 30$ at the 1% significance level. You conclude
A) that $H_0$ should be rejected
B) that $H_0$ should not be rejected
C) that $H_a$ should be accepted
D) this is a borderline case and no decision should be made

Problem 15:
The time needed for college students to complete a certain paper-and-pencil maze follows a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the mean time $\mu$ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the maze. You compute the average time $\bar{x}$ that it takes these students to complete the maze and test the hypotheses $H_0: \mu = 30$ $H_a: \mu \neq 30$ You find that the results are significant at the 5% significance level. You may also conclude
A) the test would also be significant at the 10% level
B) the test would also be significant at the 1% level
C) both of the above
D) none of the above
Problem 16
Suppose, over a long period of time in which the posted speed limit was 65 mph, the proportion of drivers who drive at least 10 miles over the speed limit was well established as an average of 12.3% a day. After the speed limit dropped to 55 mph, the proportion of drivers who drive at least 10 miles over the speed limit in a sample of size 100 was found to be 13.6%. Does the lower speed limit cause an increase in the number of drivers who do not respect the speed limit, or could it be only due to sampling variability? Compute a statistical test.

Problem 17
After once again losing a football game to the college’s arch rival, the alumni association conducted a survey to see if alumni were in favor of firing the coach. An SRS of 100 alumni from the population of all living alumni was taken. Sixty-four of the alumni in the sample were in favor of firing the coach. Let $p$ represent the proportion of all living alumni who favor firing the coach. A 95% confidence interval for $p$ is

A) $(0.63-0.047, 0.63+0.047)$
B) $(0.63-0.078, 0.63+0.078)$
C) $(0.63-0.093, 0.63+0.093)$
D) $(0.63-0.128, 0.63+0.128)$

Problem 18
Referring to the previous question, suppose you wished to see if the majority of alumni are in favor of firing the coach. To do this you test the hypotheses

$H_0$: $p = 0.50$
$H_a$: $p > 0.50$

The p-value of the test is

A) between .05 and .1
B) between .01 and .05
C) between .001 and .01
D) below 0.001