

Paired-sample t -test

1. Assumptions

- **Experimental Design:** The sample forms two treatment groups, where each subject in one group is paired with a subject in the other group.
- **Null Hypothesis:** The average of the pair differences is not significantly different than zero.
- **Population Distribution:** Each treatment group is approximately normal.
- **Sample Size:** The sample sizes of the treatment groups are arbitrary.

2. Inputs for the t -test

- Sample size: n
- Sample mean: \bar{d} of the differences, where $d_i = x_{1i} - x_{2i}$.
 $x_{1i} \in \text{Treatment Group 1}; x_{2i} \in \text{Treatment Group 2}$.
- Sample standard deviation: s_x (SD of the differences)
- Standard error of mean: $\text{SE}_{\text{mean}} = \frac{s_d}{\sqrt{n}}$
- Null hypothesis value: c
- The level of the test: α

3. Five Steps for Performing the Test of Hypothesis

1. State null and alternative hypotheses:

$$H_0 : \mu = 0, \quad H_1 : \mu \neq 0$$

2. Compute test statistic:

$$z = \frac{\bar{d} - \mu}{\text{SE}_{\text{mean}}},$$

assuming the null hypothesis value 0 for μ .

3. Compute $100(1 - \alpha)\%$ confidence interval I for t , using the t -table with $n - 1$ degrees of freedom.
4. If $z \in I$, accept H_0 ; if $t \notin I$, reject H_1 and accept H_0 .
5. Compute p -value with statistical software.

4. Discussion

If both of the treatment groups are normally distributed, then the differences $d_i = x_{2i} - x_{1i}$ are also normally distributed, so the assumptions of the one-sample t -test apply for the differences.

4. A Sample Problem

From Mendenhall and Sincich, p. 57: To measure whether a new method for teaching reading works better than the standard method, eight pairs of children that have had difficulties learning how to read are located. Each pair is arranged to contain two children with similar IQ scores. Then one child out of each pair, designated as Group 1, is taught using the new teaching method, the other child out of the pair, designated as Group 2, is taught using the standard teaching method. The difference score for each pair is computed: $d_i = x_{1i} - x_{2i}$. Perform a 0.01-level paired-sample t -test to see if there is a significant difference between the two teaching methods. Here are the inputs:

$$n = 8 \quad \bar{d} = 4.375 \quad s_d = 1.685 \quad c = 0 \quad \alpha = 0.01.$$

$$SE_{\text{mean}} = 0.596 = \frac{s_x}{\sqrt{n}} = \frac{1.685}{\sqrt{8}} = 0.596.$$

The five steps of the t -test:

1. State the null and alternative hypotheses:

$$H_0 : \mu_d = 0, \quad H_1 : \mu_d \neq 0$$

2. Compute the test statistic:

$$z = \frac{\bar{d} - \mu}{SE_{\text{mean}}} = \frac{4.375 - 0}{0.596} = 7.34$$

3. Compute a $100(1 - \alpha)\% = 99\%$ confidence interval I : use the t -table with $n - 1 = 8 - 1 = 7$ degrees of freedom to show that $[-3.499, 3.499]$ is a 99% confidence interval for t .
4. Determine whether to accept or reject H_0 : $7.34 \notin [-3.499, 3.499]$, so reject H_0 .
5. Let SAS or R compute the p -value: 0.0002.