Statistics of the Geometry of Object Populations

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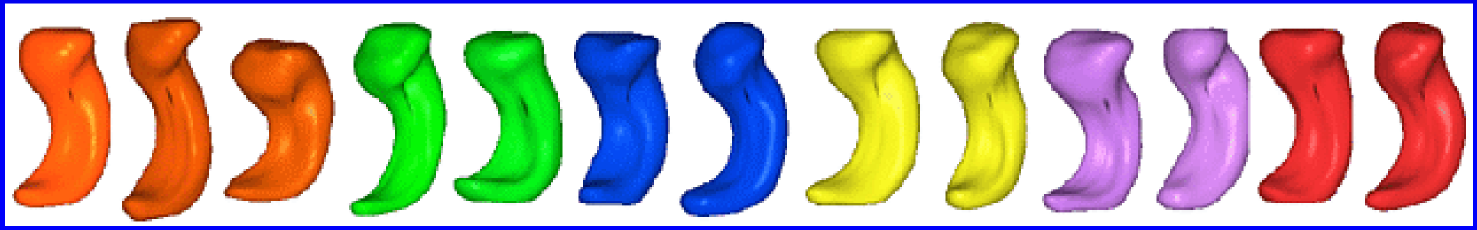
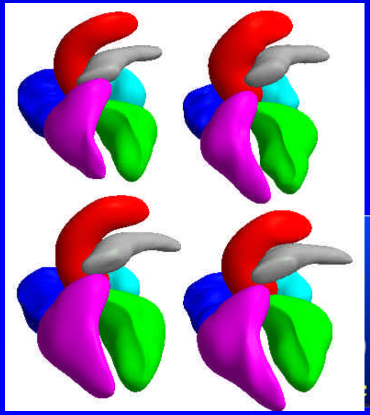
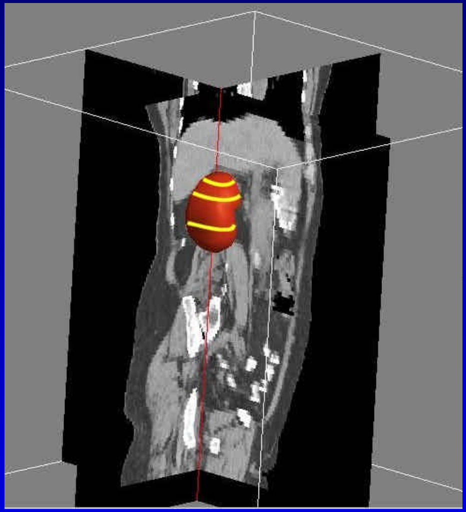
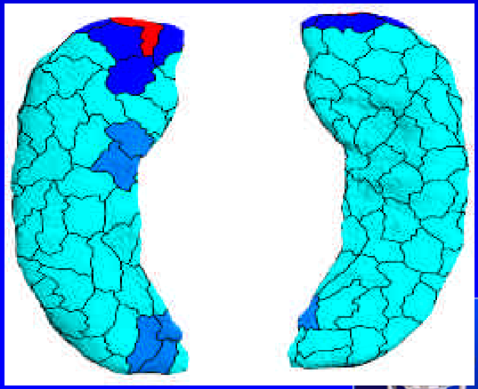




Need Probability p(**z**) of Objects in Populations with **z** =Deformation(Mean)

- Let **z** be transformation from mean, localized into object sections
- Uses for p(**z**)
  - Sampling p(**z**) to communicate anatomic variability in atlases
  - Log prior in posterior optimizing deformable model segmentation
    - Optimize log p(**z**) + log p(I|**z**)
  - Compare two populations
    - Medical science: localities where p(**z**|healthy) & p(**z**|diseased) differ
    - Diagnostic: Is particular patient's geometry diseased? p(**z**|healthy, I) vs. p(**z**|diseased, I)





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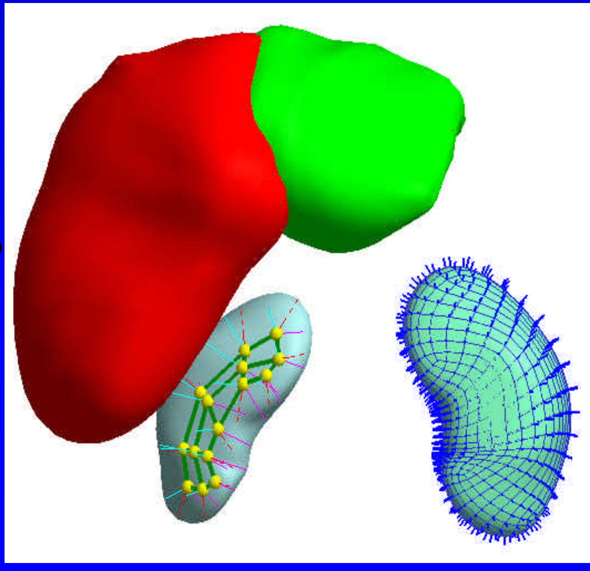
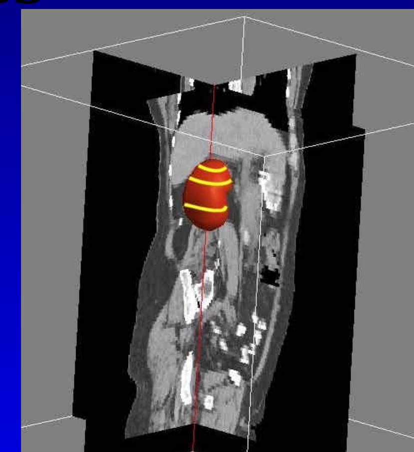
Needs of Probability Representation p(z)

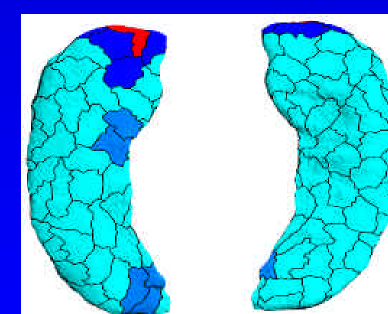
- Accurate estimation of p(z) using limited samples
  - Positional correspondence
  - Multiple scale levels (beat HDLSS), incl. obj. ensembles
- Rich and intuitive characterization of geometric effects
  - Local translations, but also local rotations, magnifications
- Null probabilities for geometrically illegal objects
  - Statistics on geometric transformations
- Localization
  - Residues in multiscale framework



Represent the Egg, not the Eggshell

- The eggshell: object boundary primitives
  - B-reps
  - Transformations of primitives: local displacement
- The egg: object interior primitives
  - M-reps
  - Transformations of primitives: local displacement, local bending & twisting (rotations), local swelling/contraction





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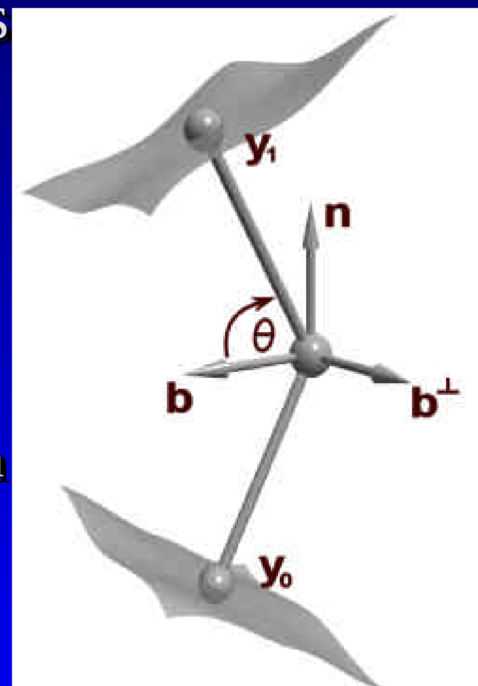
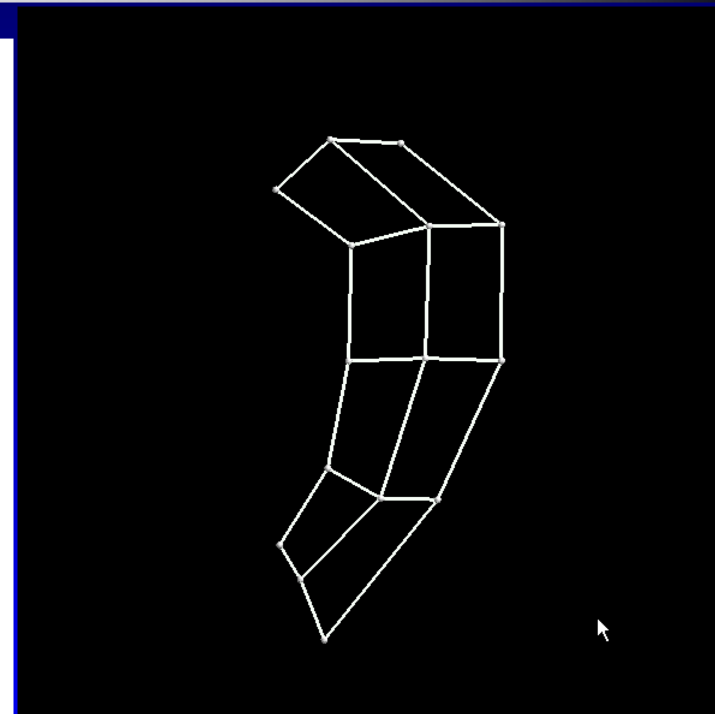
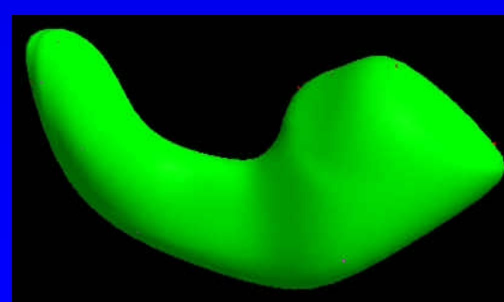
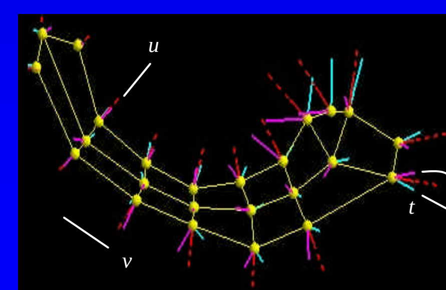
Geometric power of medial atom as basis for describing a geometric transformation

- Medial atom parameters carry position, metric, two orientations
  - $T \in \Re^3 \times \Re^+ \times S^2 \times S^2$
    - From base atom: Hub translation × Spoke magnification in common × spoke_1 rotation × spoke_2 rotation
    - $S^2 = SO(3)/SO(2)$
- M-rep is n-tuple of medial atoms
  - T^n, n local T's, a symmetric space



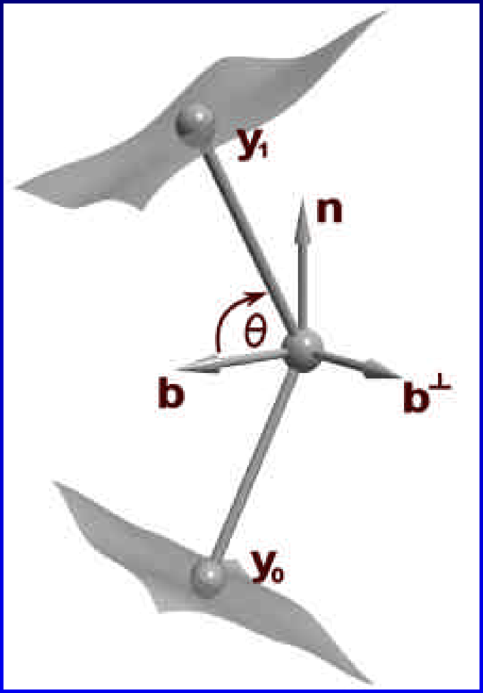


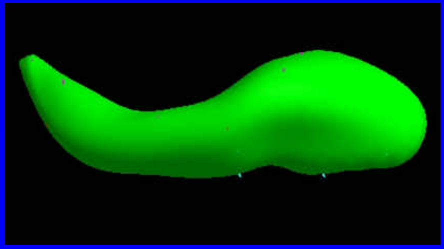


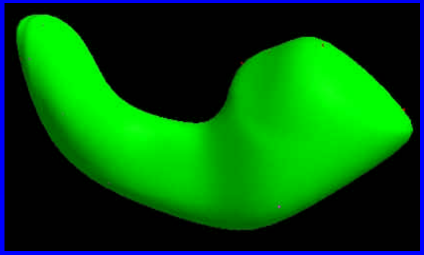


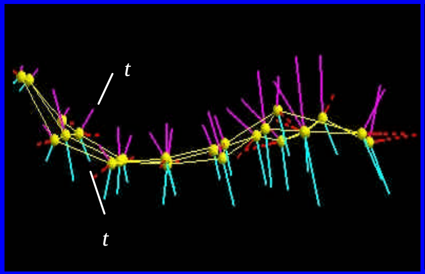
Medial atom or m-rep as a geometric transformation, a point on a symmetric space

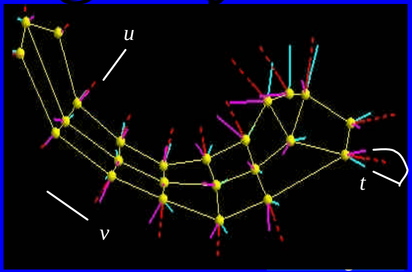
- $T \in \Re^3 \times \Re^+ \times S^2 \times S^2$
- Some m-rep parameters are not linear
  - Rotations
  - Magnifications
- High-dimensional, curved space:
  - Quotients of Lie groups
- Standard linear statistics do not achieve legality





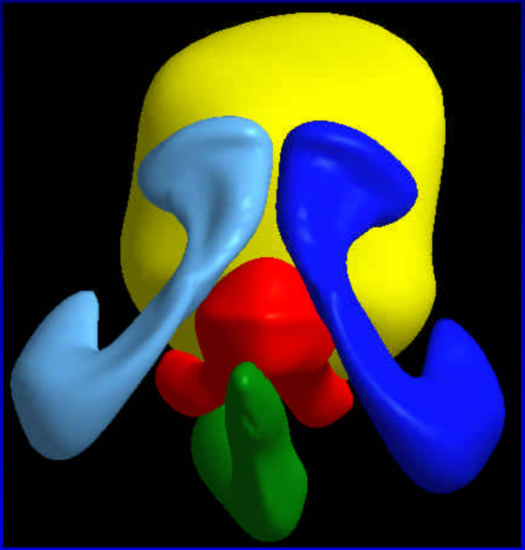




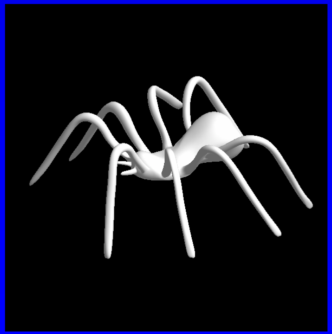


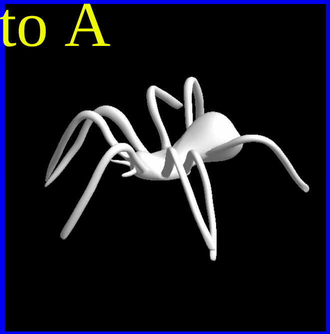
M-reps as Points in Curved Space, Geodesics

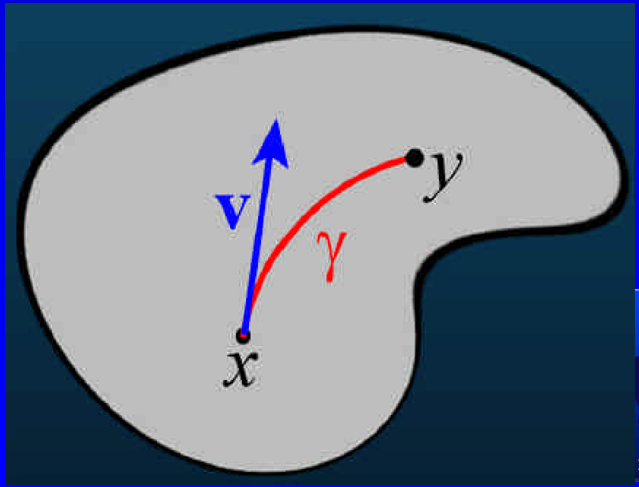
- Deformation as **paths** of local rotation, swelling, & displacement
  - i.e., on symmetric space’s surface
  - Geodesics: minimum energy
    - Geometric
    - Physical
- E.g., tweening from key frames in animation



A to B to A to C to A







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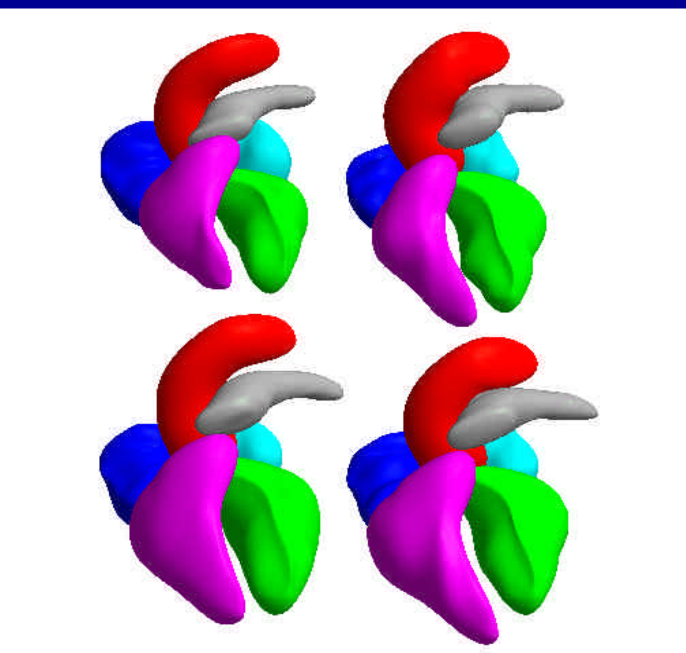
Animation - Demo

Principal Geodesic Analysis [Fletcher] - go to Fletcher presentation

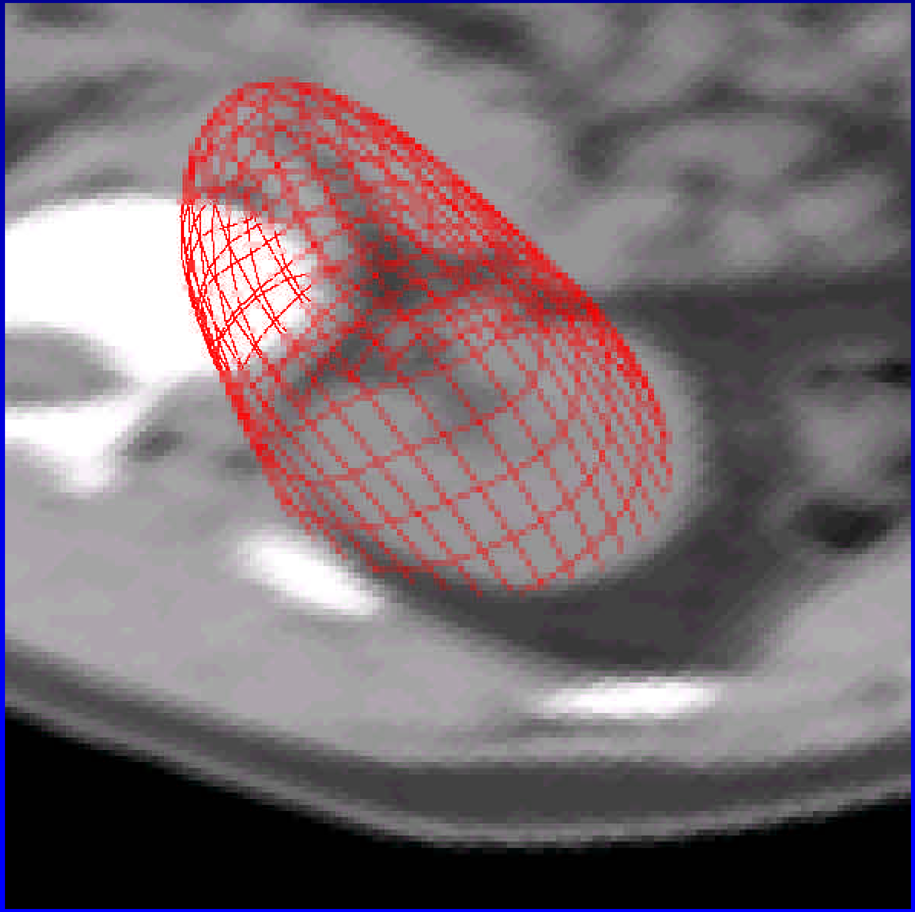


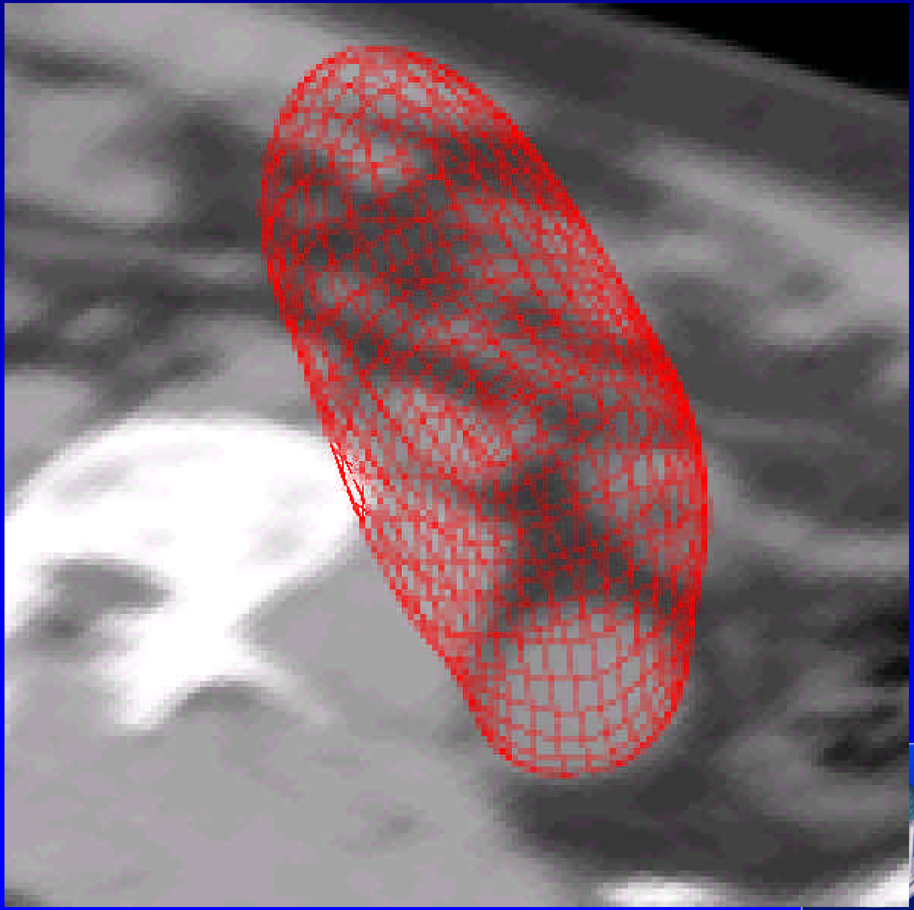
Example: 4 heart models, each made up of 7 objects [Pilgram]

- Calculated via tangent hyperplanes through mean and “exponential” & “logarithmic maps”
- Demo for object ensemble, for object

Segmentation of a Kidney by Deformable M-reps (~10 sec.)

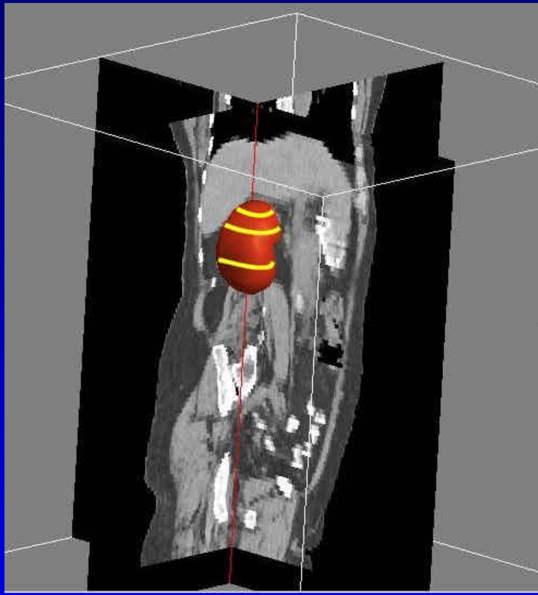
Optimize $p(\underline{\mathbf{z}}|\underline{I})$, thus $\log p(\underline{\mathbf{z}}) + \log p(\underline{I}|\underline{\mathbf{z}})$

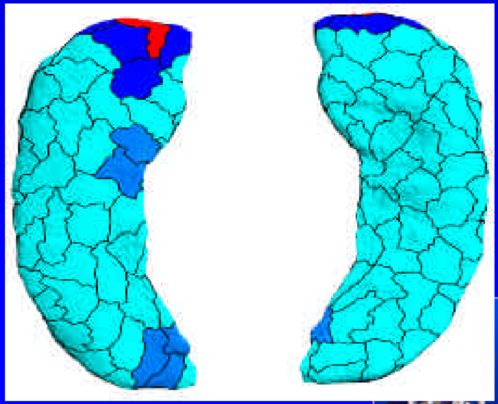




Efficiency is Critical for Objects

- Typically more than 10^4 primitives
- Time efficiency
  - Optimizing probability
  - Simulated physical behavior (e.g., motion)
- Statistical efficiency: number of training samples needed
  - Inter-sample spatial correspondence
  - Rich local geometric transformations
- Both efficiencies require multiple scales (levels of locality)
  - O(N) not O(N^2)





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Scale Situations in Various Sampled Geometric Analysis Approaches

Global coef for each level of detail

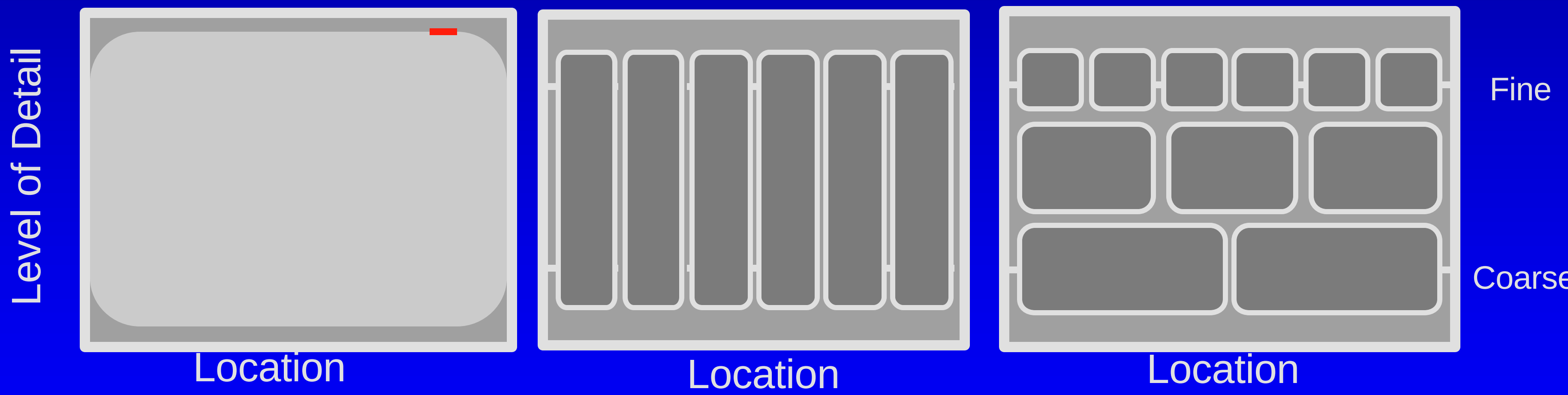
Examples: boundary spherical harmonics, global principal components

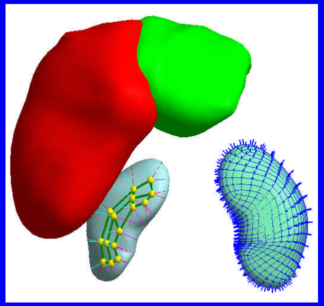
Multidetail feature

boundary points, dense position displacements, medial atoms

Detail residues

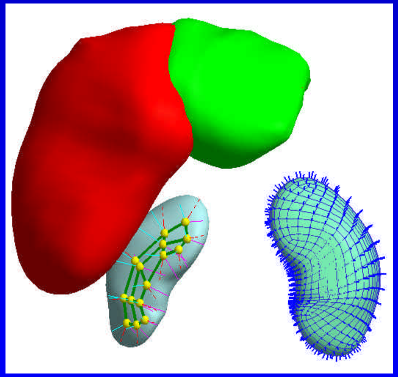
m-rep object hierarchy, wavelets

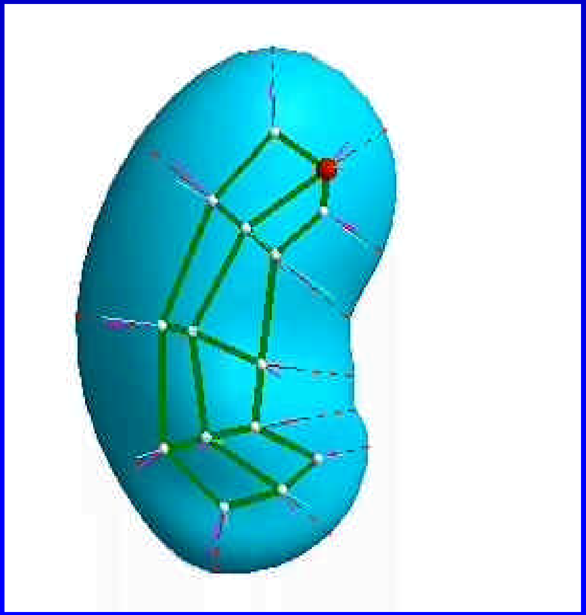




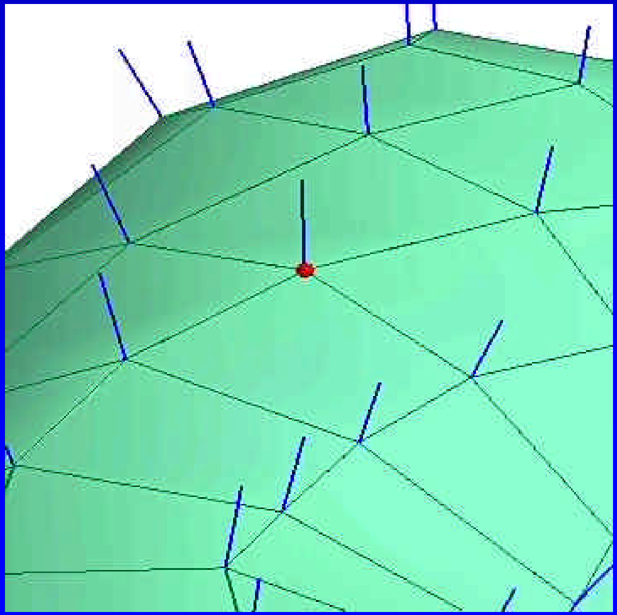
Residues at the Object, Atom and Boundary Scale Levels

- Object transformations are transformations of all the object's atoms taken as a group
- Atom residues are $\in \Re^3 \times \Re^+ \times S^2 \times S^2$
- Boundary residues (part of m-rep) are normalized displacements from medially implied boundary positions





atom residue



boundary residue

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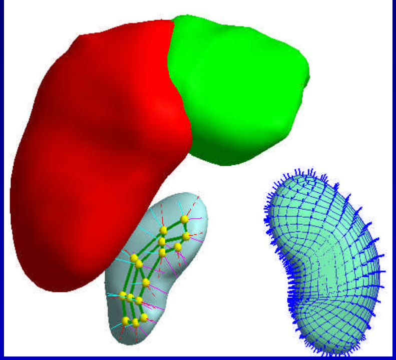
Markov Random Field Models for Primitives with Neighbor Relations

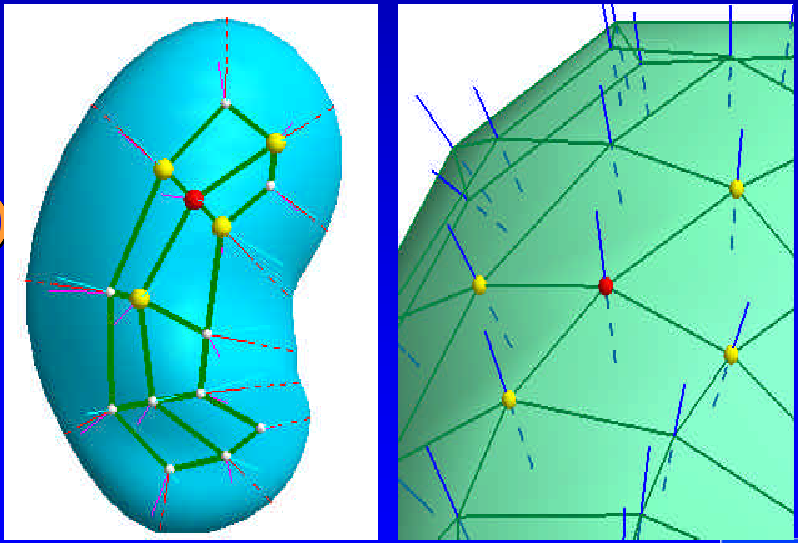
- Markov assumptions
  - Inter-scale residues ($\Delta T^m = T^m - T^{m-1}$ models the residue at scale level m)

    $\text{Prob}(T^m | \{T^s, s<m\})$

    $= \text{Prob}(T^m | T^{m-1}) = \text{Prob}(T^m - T^{m-1})$
  - At entity i,model $\text{Prob}(\Delta T^m_i)$, where

    $T^m_i = \text{Pred}(T^m_{\{N(i,j)\}}) + \Delta T^m_{(i,j)}$
  - Both have moderate dimension



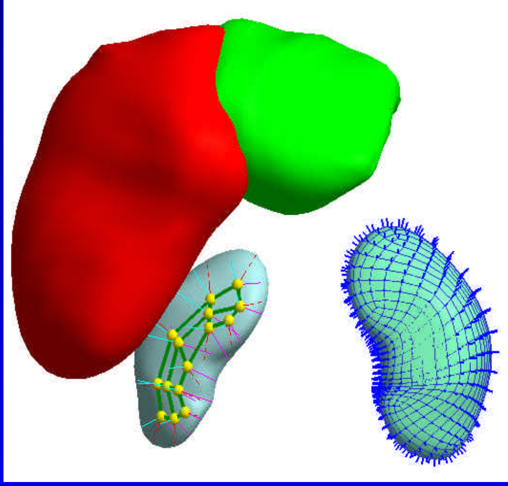


atom level boundary level

quad-mesh neighbor relations

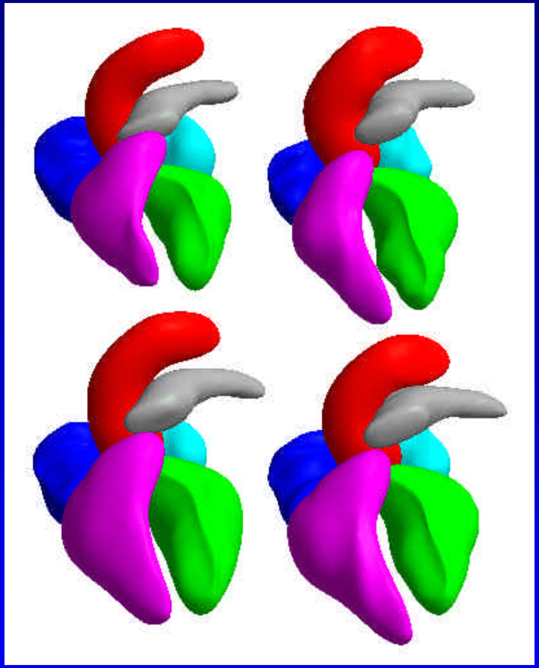
Deformation Parameters by Scale Level for M-reps

Object ensemble	Sim. Transf & PGC coef's
Object	Sim. Transf & PGC coef's
Main figure	Sim. Transf & PGC coef's
Subfigure	Sim. Transf. in host (u,v,t) & PGC coef's
Medial atom	Atom parameters
Boundary vertex	Displacement along normal (medial atom spoke)



Training M-rep Probabilities [Fletcher, Han, Dam]

- Start with
  - N binary images of segmentations
  - # of figures and medial atom grid size of each figure
- Fit m-reps to images
- Regularize m-reps and refit
- Align using Lie group distances
- Compute mean and principal geodesics at top scale
- Fit PGCs to cases and refit residues



A seven-object heart
(in future 3 multifigure objects: pericardium, right, left)

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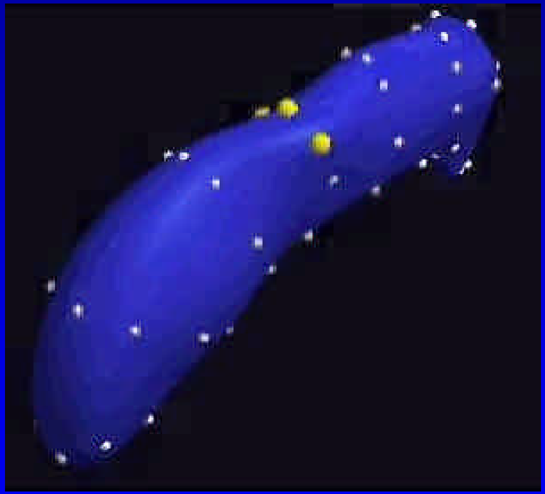
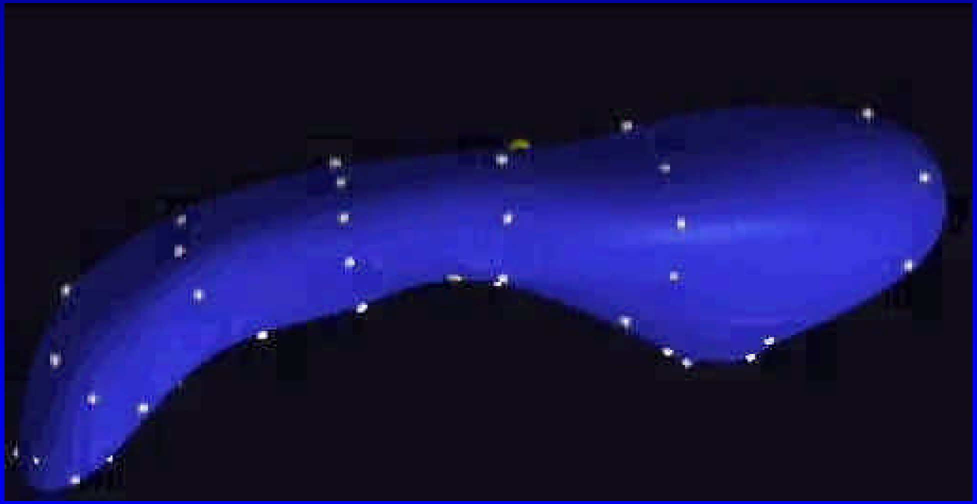
Hippocampus medial atom (residue) statistics ([Lu]

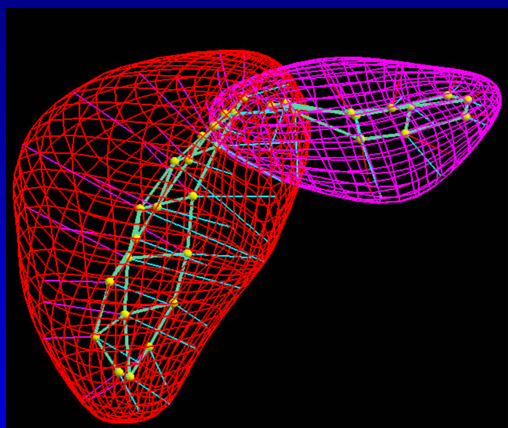
- Global over hippocampus vs. by locality
- Residue
  - From next larger scale (vs. from neighbors)



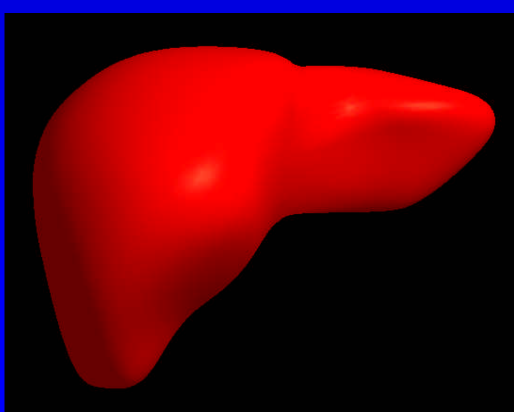


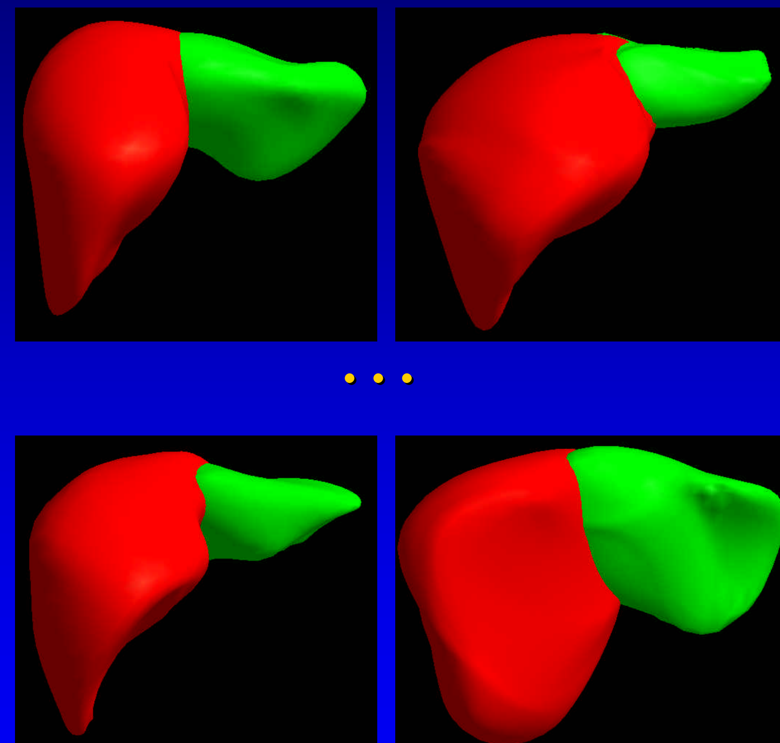
- Demo

Mean and PGCs for Liver [Lu]



Mean liver model

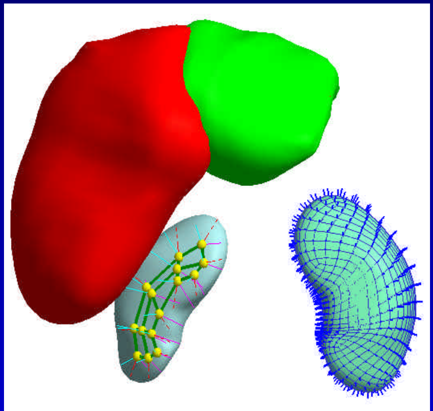




Principal modes

Our Approach to Statistical Analysis of Object Geometry

- Use m-reps:medial representation together with boundary displacement
  - primitive transformations decomposable into translations, rotations, magnifications



[Pizer, IJCV 2003 and Joshi, TMI 2002]

- Build Markov random field models for residues
  - inter-scale residues between scale levels
  - intra-scale residues among neighboring primitives at each scale level

[Lu, Scale Space 2003]

- Residue models of geometric transformations and the metrics on them are defined and analyzed via geodesics on symmetric space [Fletcher, CVPR & IPMI 2003]

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Other Applications of Lie Group and Lie Group Quotient Statistics

- Statistics of tensors (from DTI) [Fletcher]
- Statistics of neural fiber bundle geometry [Gerig]
- Statistics of joystick gestures [Feasel]
- Extension to diffeomorphism group
  - Incl. as small scale residue within m-reps
- Any statistical pattern recognition method, previously on linear space
  - Discrimination
    - Fisher linear discriminant
    - Support vector machines
    - Kernel methods
    - Feature selection with locality [Yushkevich]
  - Clustering

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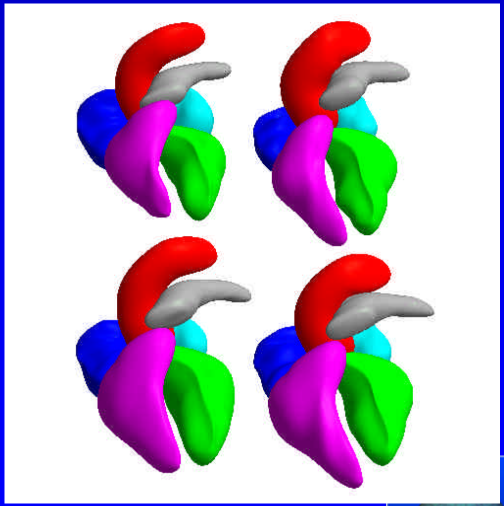
Future Work

- MRF models at other scale levels
  ⇒ complete probabilistic model on all scale levels
- Test adequacy of close-neighbor Markov assumption
- Connect image and geometric scale spaces
- Localized feature selection
  - On 3D medial
  - With multiscale residues
- Statistics of multiple multifigure objects



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For background to this talk see tutorial at website:

midag.cs.unc.edu/projects/object-shape/tutorial/index.htm

http://midag.cs.unc.edu/pubs/presentations/Pizer_SPIE.ppt

http://midag.cs.unc.edu/pubs/presentations/Joshi_SPIE.ppt

http://midag.cs.unc.edu/pubs/presentations/Gerig_SPIE.ppt

or papers at

midag.cs.unc.edu bibliography

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